The non-abelian tensor product of a pair of groups was introduced by R. Brown and J.-L. Loday. It arises in the applications in homotopy theory of a generalized Van Kampen theorem.

Let $G$ and $H$ be groups which act on each other via automorphisms and which act on themselves via conjugation. The actions are said to be compatible if $^h g' = g'(h^{-1}g')$ and $^h h' = h(h^{-1}h')$ for all $g, g' \in G$ and $h, h' \in H$. The nonabelian tensor product $G \otimes H$ is defined provided $G$ and $H$ act compatibly. In such a case $G \otimes H$ is the group generated by the symbols $g \otimes h$ with relations $gg' \otimes h = (g \otimes g')(g \otimes h)$ and $g \otimes hh' = (g \otimes h)(g \otimes h')$ for $g, g' \in G$ and $h, h' \in H$.

If $G = H$, we call $G \otimes G$ the tensor square of $G$. Here the action is conjugation which is always compatible. Good progress has been made in determining the nonabelian tensor square for large classes of groups.

However in the case of nonabelian tensor products the enigma of compatible actions has prevented such progress. Only in a few cases the nonabelian tensor product of two groups with nontrivial compatible actions has been determined. One such case is the nonabelian tensor product of two infinite cyclic groups, where the mutual actions are inversion. In 1989 Gilbert and Higgins showed that the nonabelian tensor product was isomorphic to the free abelian group of rank 2, contradicting an earlier conjecture that the nonabelian tensor product of two cyclic groups is cyclic.

We were able to show that the minimal number of generators of a nonabelian tensor product of two cyclic groups does not exceed two. Furthermore, we established a necessary and sufficient condition that a pair of actions on two cyclic groups is a compatible pair. With its help we classified all compatible actions in the case of cyclic $p$-groups. The resulting nonabelian tensor products turn out to be cyclic $p$-groups with the exception of some $2$-groups with certain actions of order 2.

This is joint work with M.P. Visscher and M.S. Mohamad.

Refreshments at 2:30pm in RT 1517