Mathematics Colloquium, CSU
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3PM–4PM in RT 1516

The invariants of $n$ points on the line, and spatial polygons.

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Abstract. There are various constructions that yield a compactification of the moduli space of configurations of $n$ distinct labeled points on the Riemann sphere modulo Mobius transformations. For example if $n = 4$, the cross-ratio of 4 distinct points can take on any value other than 0 or infinity, and it completely determines the equivalence class of the 4 points modulo Mobius transformations. Hence the (non-compactified) moduli space is identical to the sphere minus two points (the points 0 and infinity from the Riemann sphere). To compactify, it is natural to consider configurations of 4 points where the cross-ratio is still well-defined (i.e. not zero/zero) but can take on these additional values of 0 and infinity (in which case the moduli space would then be the full Riemann sphere). This can be accomplished by allowing pairs of points to collide, but such that no three collide.

Geometric Invariant Theory, due to David Mumford, is a general method by which one may compactify such moduli spaces. We shall study the simple case of $n$ points on the Riemann sphere (for $n \geq 4$). We will describe the GIT (Geometric Invariant Theory) quotient $(\mathbb{P}^1)^n / \text{PGL}(2)$, explaining how it is embedded into projective space by describing its equations, and we will explain how its points coincide with $n$-gons in Euclidean space (with labeled edges having prescribed lengths) modulo rotations and translations. (This is based on joint work with J. Millson, A. Snowden, and R. Vakil, spanning several years.)

* Refreshments at 2:30 PM in RT 1517